

Bayesian Election Prediction: BJP vs Congress

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1 Problem Statement

1.1 What Are We Solving?

We aim to predict the election outcome in a village of 250 voters between two political parties:

- **BJP**: Bharatiya Janata Party
- **Congress**: Indian National Congress

Given:

- Limited survey data (20 voters)
- Historical election patterns
- Village size: 250 voters

Our goal is to compute:

1. Probability of BJP winning
2. Probability of Congress winning
3. Uncertainty in predictions

1.2 Why Bayesian Approach?

Traditional polling has limitations:

- Small samples create large margins of error
- Historical context is ignored
- Uncertainty isn't quantified

Bayesian methods solve these by:

- Incorporating prior knowledge
- Updating beliefs with new evidence
- Providing probabilistic forecasts

2 Solution Approach: Bayes' Theorem

We use the fundamental Bayesian update rule:

$$\underbrace{P(\text{Party}|\text{Survey})}_{\text{Posterior}} = \frac{\overbrace{P(\text{Survey}|\text{Party})}^{\text{Likelihood}} \cdot \overbrace{P(\text{Party})}^{\text{Prior}}}{\underbrace{P(\text{Survey})}_{\text{Evidence}}}$$

3 Step-by-Step Solution

3.1 Step 1: Define Prior Probabilities

- **What:** Initial belief before seeing survey data
- **Why:** Incorporates historical election results
- **How:** Based on past village elections

$$\begin{aligned} P(\text{BJP}) &= 0.6 \quad (\text{BJP won 60\% of similar elections}) \\ P(\text{Congress}) &= 0.4 \quad (\text{Congress won 40\%}) \end{aligned}$$

3.2 Step 2: Gather New Evidence (Likelihood)

- **What:** Survey of 20 randomly selected voters
- **Results:** 12 support BJP, 8 support Congress
- **Why:** Measures how probable the survey is under each hypothesis

Likelihood calculations:

$$\begin{aligned} P(\text{Survey}|\text{BJP}) &= \binom{20}{12} (0.6)^{12} (0.4)^8 \quad (\text{If BJP truly has 60\% support}) \\ P(\text{Survey}|\text{Congress}) &= \binom{20}{12} (0.4)^{12} (0.6)^8 \quad (\text{If Congress truly has 40\% support}) \end{aligned}$$

3.3 Step 3: Apply Bayes' Theorem

- **What:** Combine prior and likelihood
- **Why:** To update our beliefs with new evidence
- **How:** Compute posterior probabilities

First, calculate normalizing constant:

$$P(\text{Survey}) = (0.18 \times 0.6) + (0.12 \times 0.4) = 0.156$$

Then update beliefs:

$$P(\text{BJP}|\text{Survey}) = \frac{0.18 \times 0.6}{0.156} \approx 0.73$$

$$P(\text{Congress}|\text{Survey}) = \frac{0.12 \times 0.4}{0.156} \approx 0.27$$

3.4 Step 4: Extrapolate to Full Village

- **What:** Predict actual election outcome
- **Why:** Survey sample (20) \neq Village size (250)
- **How:** Model votes as binomial distributions

$$\text{BJP votes} \sim \text{Binomial}(250, 0.6)$$

$$\text{Congress votes} \sim \text{Binomial}(250, 0.4)$$

Probability BJP wins (BJP votes $>$ Congress votes):

$$P(X_{\text{BJP}} \geq 126) \quad (\text{since } 126 > 125)$$

Using normal approximation:

$$\mu = 250 \times 0.6 = 150$$

$$\sigma = \sqrt{250 \times 0.6 \times 0.4} \approx 7.75$$

$$P(\text{BJP wins}) \approx P\left(Z \geq \frac{126 - 150}{7.75}\right) = P(Z \geq -3.1) \approx 0.999$$

4 Interpretation of Results

	BJP	Congress
Probability of being preferred	73%	27%
Probability of winning election	99%	1%

Table 1: Key Differences in Prediction Types

4.1 Why Two Different Probabilities?

- **73% vs 99%:** The first measures which party *voters prefer* based on survey, the second measures which party will *win the election* given voting mechanics
- **Crucial insight:** Even a modest preference (60-40) leads to near-certain victory in large populations

5 Limitations and Improvements

5.1 Potential Biases

- Sampling bias: Survey may not represent village
- Prior misspecification: Historical data may not apply
- Non-random voting: Social influences not modeled

5.2 Enhancement Strategies

Method	Benefit
Larger survey sample	Reduces margin of error
Hierarchical modeling	Accounts for village differences
Sensitivity analysis	Tests prior assumption robustness